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# REDUCTION OF PERPENDICULAR TO GRAIN STRESS IN DOUBLE-TAPERED GLULAM BEAMS WITH THE TRANSVERSE REINFORCEMENT

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In the present paper tensile stresses perpendicular to the grain in reinforced double-tapered beams made of glued laminated timber are discussed. The beams are analysed using the finite element method within the linear elasticity theory with the influence of orthotropic material properties. The main objective is to assess the influence of transverse reinforcement on the values and distributions of the analysed stresses and to identify the most efficient scheme of reinforcement. The obtained results prove that, with the use of the proposed tools, it is possible to assess the level of stress, including delaminating stress, and to indicate the areas of occurrence of such stress with high precision.

*Keywords*: glulam, double-tapered beam, transverse reinforcement

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# 1. INTRODUCTION

Double-tapered beams are nowadays one of the most commonly used structural elements made of glued laminated timber. They are often used as roof girders [1], as their geometry naturally provides a desired roof inclination. Furthermore, the material of such beams is used with better efficiency, because the geometry tends to reflect the bending stress distribution under most load cases.

In the present paper simply supported double-tapered glulam beams are considered, with a standard geometry described in Eurocode 5 [2] (Fig. 1). In the design and analysis of such beams, apart from shear and bending stresses, tensile stress perpendicular to the wood fibres should be taken into account and reinforcement against this stress should be considered if needed [1, 3-7].

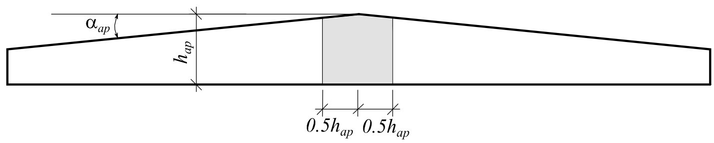



Fig. 1. Double-tapered beam considered in Eurocode 5 with the apex zone.

According to the design standard Eurocode 5 [2], it is recommended to check normal stress perpendicular to the grain in the apex zone (Fig. 1) and to design appropriate reinforcement if such stress is too big. Eurocode 5 recommends the following formula for calculation of stress perpendicular to the grain:

(1.1) $$\sigma_{\mathrm{t,90,d}} = k_{\mathrm{p}} \frac{6M_{\mathrm{ap,d}}}{bh_{\mathrm{ap}}^2},$$

where:

$M_{\mathrm{ap,d}}$ – design bending moment in the middle of the span, $b$ – beam's width, $h_{\mathrm{ap}}$ – apex height and $k_{\mathrm{p}} = 0.2\tan(\alpha_{\mathrm{ap}})$.

The maximum tensile stress perpendicular to the grain should satisfy the expression

(1.2) $$\sigma_{t,90,d} \leq k_{dis} k_{vol} f_{t,90,d},$$

where:

$f_{t,90,d}$ – design tensile strength perpendicular to grain, $k_{dis}$ – factor which takes into account the effect of stress distribution in the apex zone and for double-tapered beams equals 1.4, $k_{vol}$ – volume factor given by the formula $k_{vol} = (V_0/V)^{0.2}$, where: $V_0$ – reference volume of 0.01 m$^3$, $V$ – volume of the apex zone (see Fig. 1) which should not be taken greater than $2V_b/3$, with $V_b$ – total volume of the beam.

If the expression (1.2) is not satisfied or the designer's intention is to strengthen the beam, reinforcement of the apex zone should be designed. It shall fulfil the following condition:

(1.3) $$F_{t,90,d} \leq R_{ax,Rd},$$

where:

$F_{t,90,d}$ – design force in the connector, $R_{ax,Rd}$ – design load-carrying capacity of an axially loaded connection.

The force acting on the connector is calculated as

(1.4) $$F_{t,90,d} = \frac{\sigma_{t,90,d} b a_1}{n},$$

where:

$a_1$ – connector's spacing along the grain, $n$ – number of rows.

Design load-carrying capacity of an axially loaded connection should be calculated as a minimum value of the load-carrying capacity of timber and steel:

$$R_{\mathrm{ax,d}} = \min \begin{cases} k_{\mathrm{mod}} \dfrac{R_{\mathrm{ax,k}}}{\gamma_{\mathrm{M1}}} \\ \dfrac{R_{\mathrm{tens,k}}}{\gamma_{\mathrm{M2}}} \end{cases}, \tag{1.5}$$

where:

$R_{\mathrm{ax,k}}$ – characteristic load-carrying capacity of an axially loaded connection, $R_{\mathrm{tens,k}}$ – characteristic load-carrying capacity of steel, $k_{\mathrm{mod}}$ – modification factor for duration of load and moisture content, $\gamma_{\mathrm{M1}} = 1.3$ – partial factor for a material property for timber connections, $\gamma_{\mathrm{M2}} = 1.25$ – partial factor for a material property for steel.

The selection of the number of reinforcing bars, their arrangement and spacing depend on the designer. There are no quantitative studies in the literature describing how much the stresses in the beam are reduced when it is strengthened.

Although Eurocode 5 [2] gives a series of formulas for calculation of stress perpendicular to the grain, the distribution of this stress is approximated in the design standard and therefore, in many cases a proper verification and confirmation using a more detailed analysis is required.

A numerical analysis using the finite element method for two specific types of loading is presented in [8]. Wider parametric analyses taking into account geometrical (slope angle – apex height) and physical parameters (orthotropic parameters of timber) are presented in [9-10]. In the paper [11] a beam with a non-standard geometry and real loading was analysed. In [12] a model that takes into account behaviour of the reinforcement within 2D elasticity theory was proposed. The papers mentioned above justify the use of the finite element method for calculation of stresses in beams with orthotropic material properties.

The present paper focuses on the analysis of stress perpendicular to the grain in a double-tapered beam with the reinforcement in comparison to the structure without reinforcement. The paper closes the gap of quantitative analysis of reinforced glulam tapered beams. The beam is analysed using the finite element method within the linear elasticity theory with the influence of orthotropic material properties. The main objective is to assess the influence of transverse reinforcement on the values and distributions of the analysed stresses and to identify the most efficient scheme of reinforcement.

# 2. DOUBLE-TAPERED BEAM WITHOUT REINFORCEMENT

The analysed double-tapered beam (Fig. 2) is a simply supported structure with a span of L=36 m, height over the supports 1 m, apex height 3.4 m and width 0.3 m. The beam is loaded with self-weight and 17 concentrated forces P=125 kN applied to its upper surface. It is made of an orthotropic material corresponding to glulam GL28h [13], with the following parameters: $E_1$=12600 MPa, $E_2$=300 MPa, $\nu_{12}$=0.35, $G_{12}$=650 MPa.

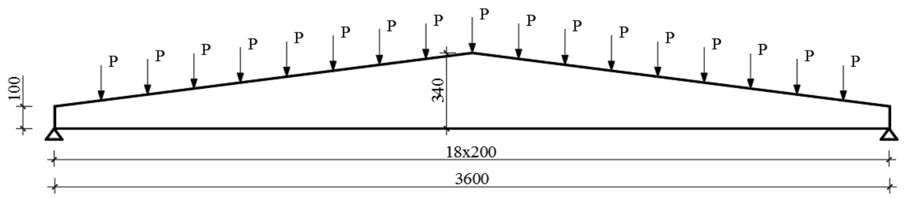



Fig. 2. Double-tapered beam – geometry, boundary conditions and loads.

The calculations were performed using the Abaqus/Standard software. Two dimensional 8-node biquadratic solid plane stress elements (CPS8R) with reduced integration were used [14]. To satisfy the finite element method convergence condition [15] a regular mesh with elements of size 0.1 m was applied. The results presented in this section concern a beam modelled as a 2D plane stress problem within the linear theory of elasticity. Similar analyses were performed on a 3D model (see chapter 4 for details). However, the distributions of the considered stress, as well as normal stress along the grain, do not show any variations in the horizontal direction perpendicular to the grain and therefore, the authors have decided to present a simpler 2D model. The differences between 3D and the corresponding 2D model of the beam are as follows: normal stress along the grain $S_{11}$ – approx. 3 %, shear stress $S_{12}$ – approx. 2 %, normal stress perpendicular to the grain $S_{22}$ – approx. 6 %, vertical deflection – approx. 3 %, where bigger values were obtained in the 3D model.

Distribution of normal stress along the grain $S_{11}$, shear stress $S_{12}$ as well as positive tensile stress $S_{22}$ perpendicular to the grain in the double-tapered beam without reinforcement is presented in Fig. 3-5.

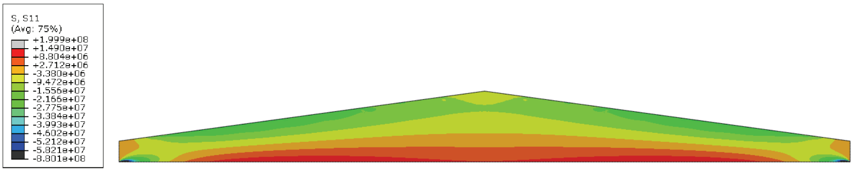



Fig. 3. Distribution of normal stress along the grain $S_{11}$ in the beam without reinforcement.

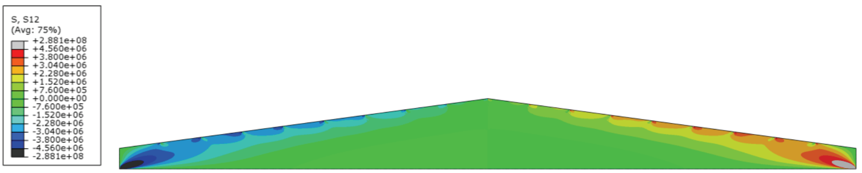



Fig. 4. Distribution of shear stress $S_{12}$ in the beam without reinforcement.

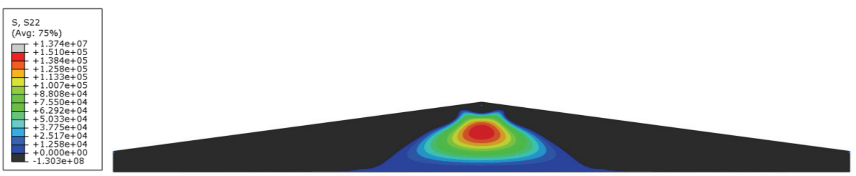



Fig. 5. Distribution of positive normal stress perpendicular to the grain $S_{22}$ in the beam without reinforcement.

Delaminating stresses appear in the apex zone, with the maximum value of 0.15 MPa reached in the upper part of the beam close to its central axis. The area of positive normal stress is much wider than the apex area considered in Eurocode 5. However, the values of the analysed stress outside the zone indicated in the design standard are very low compared to the tensile strength perpendicular to grain. From the formal point of view the stress of 0.15 MPa does not exceed the condition (1.2). However, if the loads were 25% higher, the reinforcement would have to be considered.

## 3. INFLUENCE OF REINFORCEMENT – 2D MODEL

In order to assess the influence of reinforcement on perpendicular to grain stress, the apex zone of the analysed double-tapered beam was reinforced with 6 bars, each 2.28 m long, with a spacing

0.74 m and a diameter varying from 9 mm to 13 mm. The bars' lengths were adopted arbitrarily, in agreement with engineering practice – it is a commonly used scheme of reinforcement in glulam beams, proposed in professional catalogues and used by the designers. The reinforcing bars were modelled as bar elements. Each of them was divided into 15 finite elements type B22 (three-node plane beam elements) [14]. The translational degrees of freedom of bar elements were consistent with the 2D model of the beam with free rotations. The finite element model of the reinforced beam is depicted in Fig. 6.

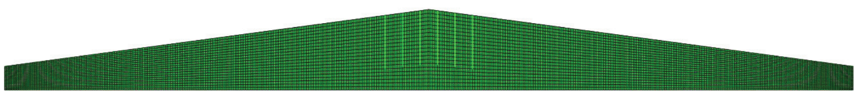


Fig. 6. Finite element model of the beam with the transverse 6 bar reinforcement.

Distributions of positive tensile stresses perpendicular to the grain in the apex zone of the beam are presented in Fig. 7. Three cases are compared: the beam without reinforcement and the beam reinforced with either 6ϕ9 or 6ϕ13 bars. It can be noticed that the applied reinforcement reduces significantly the maximum values of analysed stress in the centre of the beam and changes the stress distribution. In the case of 6ϕ13 reinforcement, the extreme values occur in the area near the bottom ends of the inner bars instead of the beam's centre.

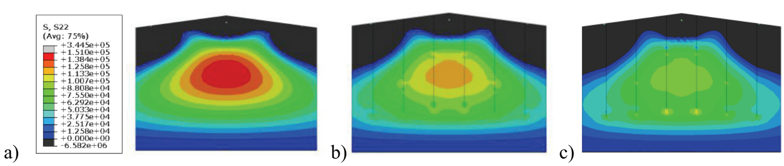



Fig. 7. Influence of reinforcement on the $S_{22}$ stress distribution: a) none; b) 6ϕ9; c) 6ϕ13.

The qualitative analysis of the influence of reinforcement on the maximum value and distribution of perpendicular to grain stress is presented in Fig. 8-9. Six cases are analysed: the beam without reinforcement and the beam reinforced with 6 bars ϕ9, ϕ10, ϕ11, ϕ12 and ϕ13.

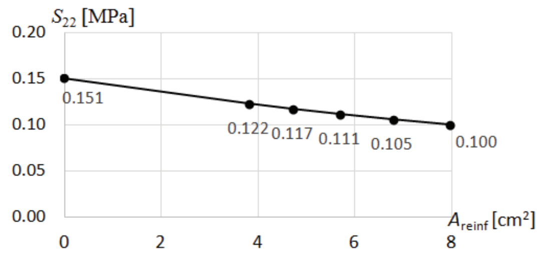



Fig. 8. Influence of the total area of reinforcement on the maximum values of perpendicular to grain stress.

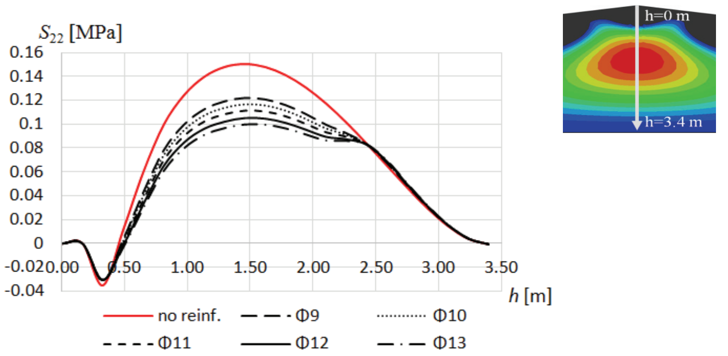



Fig. 9. Influence of 6 bar reinforcement on the distribution of perpendicular to grain stress in the apex cross-section.

The numerical simulation indicates that it is possible to reduce delaminating stress in the middle of the beam by 30% using 6ϕ13 reinforcing bars. However, special attention must be paid to the stress distribution around the ends of the bars, as the extreme values of stress might occur there. This aspect is discussed in detail in the next section of this paper, where 3D models of the beam are presented. Depending on the reinforcement used, the area and stress intensity change. A slight disturbance is visible in Fig. 9 at the height of the lower end of the reinforcement, which may, however, be a result of the adopted model.

In the performed analysis, not only the varying bars' diameters were considered, but also various reinforcement schemes. Six cases (Fig. 10) were compared: the beam without reinforcement and the beam reinforced with 2ϕ13 (two variants: inner and outer bars), 4ϕ13 (two variants: inner and outer bars) and 6ϕ13 bars. The aim of such study was to identify the most efficient location and number of reinforcing bars and to indicate the most effective scheme of reinforcement in simply supported double-tapered beams loaded with concentrated forces.

Fig. 10 shows how the distribution of positive tensile stress perpendicular to the grain in the apex zone depends on the applied reinforcement scheme. It can be noticed that the inner pair of bars (denoted by A in Fig. 11) reduces the considered stress effectively, but in the same time it induces concentrations of stresses around the bottom en  ds of reinforcement. Such concentrations can be reduced by adding another pair of bars (denoted by B in Fig. 11).

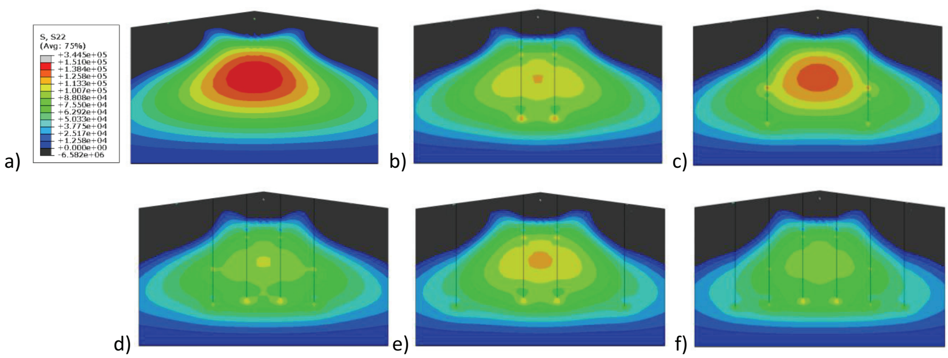



Fig. 10. Distributions of positive normal stress perpendicular to the grain $S_{22}$ in apex zones of the beams with various reinforcement schemes: a) none; b) 2ϕ13 AA; c) 2ϕ13 BB; d) 4ϕ13 BAAB; e) 4ϕ13 CAAC; f) 6ϕ13 CBAABC.

There are very small differences in stress distributions between the beam reinforced with 4ϕ13 BAAB and 6ϕ13 CBAABC, what is reflected in the results of the qualitative analysis presented in Fig. 11-12. A slight disturbance is also visible in Fig. 11 at the height of the lower end of the reinforcement. The distributions of stresses in the apex cross-section (Fig. 11) as well as the cross-section along the inner reinforcing bar (Fig. 12) indicate that the application of the outer pair of bars (denoted by C) is not necessary, as it reduces the stress values insignificantly. Another conclusion observation that can be drawn from the presented analysis is that the reinforcement should be designed as close to the centre of the beam as possible and it should be placed in the apex zone recommended in Eurocode 5.

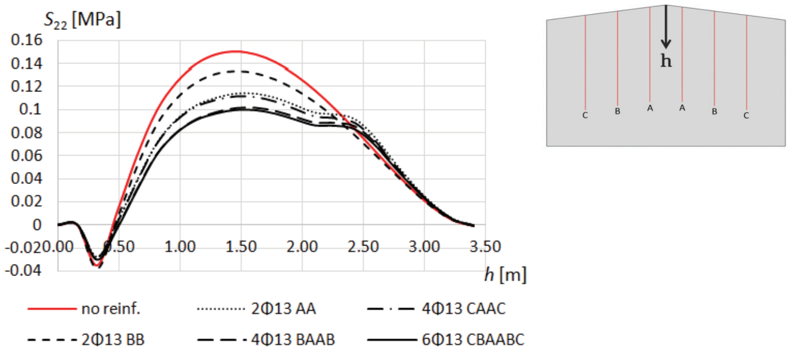



Fig. 11. Influence of reinforcement scheme on the distribution of perpendicular to grain stress in the apex cross-section.

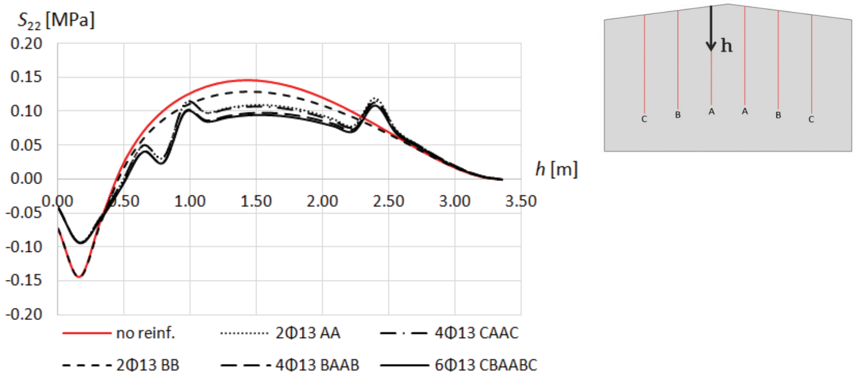



Fig. 12. Influence of reinforcement scheme on the distribution of perpendicular to grain stress in the cross-section along the inner bar.

In the cross-section shown in Fig. 12, a disturbance of the graph in a distance 0.5-1.1 m can be observed, together with the previously observed at the end of the reinforcement.

## 4. INFLUENCE OF REINFORCEMENT – 3D MODEL

A 3D numerical model of the beam was prepared in the Abaqus/Standard environment using the finite element method within the 3D linear theory of elasticity. The geometry, material parameters, boundary conditions and loading are consistent with the 2D model of the beam. 20-node quadratic brick elements (C3D20R) with reduced integration were used [14]. Mesh size is approximately

0.1 m. The reinforcement was modelled as 3-node quadratic beam elements in space (B32). Linear displacements were tied with 3D elements and the rotations of the nodes were allowed. Two cases of reinforcement distribution in the apex zone were considered (Fig. 13a, 13b): in one line (area of one bar 63.6 mm$^2$, Young's modulus 210 GPa) and in two lines (area of one bar 31.8 mm$^2$, Young's modulus 210 GPa). The modelled reinforcement corresponds to the reinforcement scheme 6ϕ9 in the 2D model.

Distribution of normal stress perpendicular to the grain in the beam with the reinforcement located in a single line is presented in Fig. 14. Delaminating stresses appear again in the apex zone, however the maximum values of 0.16 MPa are reached around the bottom ends of reinforcing bars – this is an important quantitative as well as qualitative difference compared to the 2D model of the beam (see Fig. 7b). The 3D model shows that the reinforcement placed in a single line does not reduce the maximum value of delaminating stress – it just moves to another location. Fig. 14b, 14c and 14d present distributions of stresses under consideration in the cross-sections determined by the reinforcement.

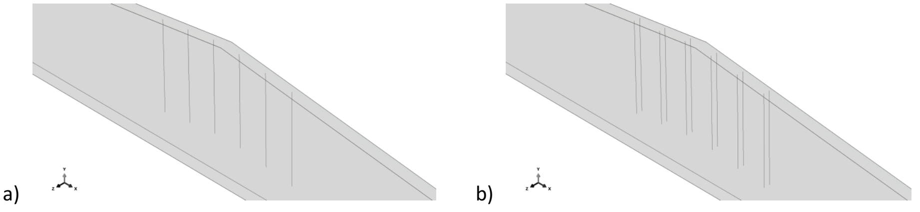
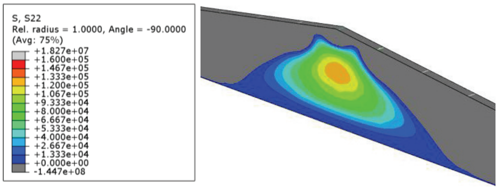
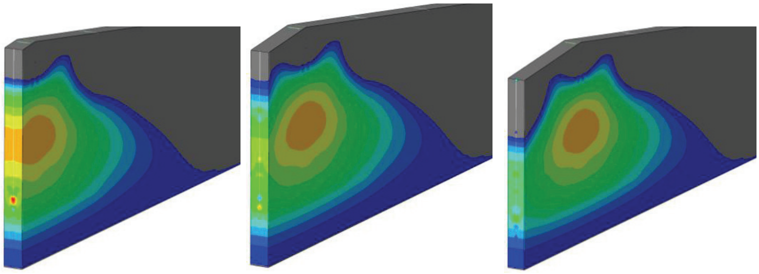



Fig. 13. 3D model of the beam: a) reinforcement in one line; b) reinforcement in two lines.

a)

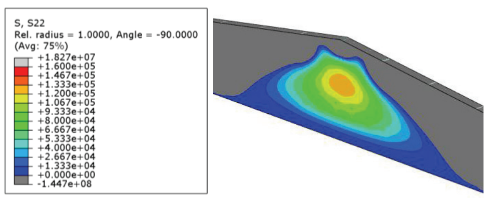




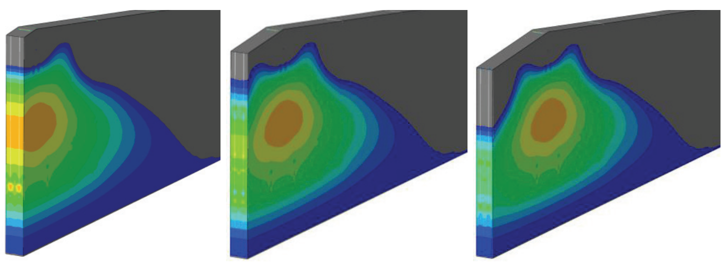
b)

Fig. 14. Stress perpendicular to grain (positive values only) with single line of reinforcement: a) general view; b) apex zone views.

Distribution of normal stress perpendicular to the grain in the beam with the reinforcement located in two lines is presented in Fig. 15.




a)



b)

Fig. 15. Stress perpendicular to grain (positive values only) with double line of reinforcement: a) general view; b) apex zone views.

It can be noticed that although the total area of reinforcement in both considered cases is the same, reinforcing bars placed in two lines reduce the stress concentrations at their bottom ends by 17 % (0.13 MPa). This phenomenon cannot be observed in the 2D model, as it does not show stress distributions in the horizontal direction across the wood fibres. Fig. 15b, 15c and 15d present distributions of stresses under consideration in the cross-sections determined by the reinforcement.

Distributions of stresses presented in Fig. 14 and 15 prove that in the case of the reinforced beams a 3D model analysis leads to the results that differ both qualitatively and quantitatively from the ones obtained in the corresponding 2D model. These results allow to justify in terms of quantity the effectiveness of the applied reinforcement.

## 5. ASSESSMENT OF RESULTS AND CONCLUDING REMARKS

The analysis carried out in this paper justifies the possibility of using the finite element method for quantitative analysis of double-tapered glued laminated beams reinforced with transverse bars. It is possible to assess the level of stress, including delaminating stress, and to indicate the areas of occurrence of these stress as precisely as possible.

To more accurately assess the impact of reinforcement on these indicators, it is proposed to introduce two quantitative parameters:

- volume of occurrence of delaminating stresses in the apex zone

$$V = \iiint\limits_{V_{S_{22}>0}} dV, \tag{5.1}$$

- the relative volume of delaminating stresses in the apex zone, which takes into account the intensity of these stresses

$$\tilde{V} = \iiint\limits_{V_{S_{22}>0}} S_{22} dV \Big/ S^{0}_{22-\max}, \tag{5.2}$$

where:

$S^{0}_{22-\max}$ – maximum delaminating stress in the beam without reinforcement.

The results are summarized in Table 1.

Table 1. Maximum stress and volume indicators.

| Model | Reinforcement | Maximum stress $S_{22-\max}$ [MPa] | Volume $V$ [m$^3$] | Relative volume $\tilde{V}$ [m$^3$] |
|---|---|---|---|---|
| 2D | No reinf. | 0.149556 | 5.81 | 1.84 |
| | 6ϕ9 CBAABC | 0.121766 | 5.68 | 1.53 |
| | 6ϕ10 CBAABC | 0.116416 | 5.61 | 1.47 |
| | 6ϕ11 CBAABC | 0.110898 | 5.59 | 1.41 |
| | 6ϕ12 CBAABC | 0.105282 | 5.55 | 1.35 |
| | 6ϕ13 CBAABC | 0.100478 | 5.49 | 1.28 |
| | 2ϕ13 AA | 0.113591 | 5.70 | 1.55 |
| | 2ϕ13 BB | 0.132692 | 5.70 | 1.61 |
| | 4ϕ13 BAAB | 0.103565 | 5.60 | 1.37 |
| | 4ϕ13 CAAC | 0.110957 | 5.62 | 1.45 |

The reference volume of the apex zone according to Fig. 1 is 5.22 m$^3$. The application of the proposed reinforcement reduces the level of delaminating stress by approx. 30 %. Bars marked as A are of key importance. It is recommended to place additional B bars. Third row of the bars C is in principle unnecessary. The volume of the delaminating stress zone in FE models is slightly larger (in particular wider) than in the standard model. However, these stresses vary and are rather small in a significant area. The volumes of the zones in reinforced and unreinforced beams are similar.

A more reliable parameter for quantitative analysis is the relative volume. The presence of reinforcement significantly reduces this volume. In the reinforced beam with 6 CBAABC bars or 4 BAAB bars, the relative volume decreases by approx. 30 %, which confirms the recommendations of the system based on the assessment of the maximum stress level. The 2D model is sufficient to quantify the impact of reinforcement on delaminating stresses. The 3D model shows the location of extreme stresses over the beam width, which is important when the reinforcement is placed in two rows.

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Rys. 14. Naprężenia prostopadłe do włókien (tylko dodatnie wartości) przy jednym rzędzie zbrojenia: widok ogólny (a), widoki stref kalenicowych (b), (c), (d).

Fig. 15. Stress perpendicular to grain (positive values only) with double line of reinforcement: general view (a), apex zone views (b), (c), (d).

Rys. 15. Naprężenia prostopadłe do włókien (tylko dodatnie wartości) przy dwóch rzędach zbrojenia: widok ogólny (a), widoki stref kalenicowych (b), (c), (d).

Tab. 1. Maximum stress and volume indicators.

Tab. 1. Wskaźniki maksymalnych naprężeń i objętości.

# REDUKCJA NAPRĘŻEŃ PROSTOPADŁYCH DO WŁÓKIEN W DWUTRAPEZOWYCH BELKACH Z DREWNA KLEJONEGO WARSTWOWO Z POPRZECZNYM ZBROJENIEM

*Słowa kluczowe*: drewno klejone warstwowo, belka dwutrapezowa, zbrojenie poprzeczne

**STRESZCZENIE:**

W niniejszej pracy skupiono się na analizie naprężeń prostopadłych do włókien w belkach dwutrapezowych z drewna klejonego warstwowo ze zbrojeniem poprzecznym. Belki dwutrapezowe są obecnie jednymi z najczęściej stosowanych elementów konstrukcyjnych z drewna klejonego. Ze względu na swój kształt, używane są powszechnie jako dźwigary dachowe. Ponadto, belki te charakteryzują się dobrym wykorzystaniem materiału, ponieważ ich geometria zwykle odzwierciedla rozkład momentów zginających od większości przypadków obciążeń.

W pracy analizowane są swobodnie podparte, dwutrapezowe belki klejone, o standardowej geometrii opisanej w normie projektowej EN-1995-1-1. W projektowaniu i analizie tego typu belek należy brać pod uwagę, obok naprężeń stycznych i normalnych, naprężenia rozciągające prostopadłe do włókien drewna, a w niektórych przypadkach trzeba tez uwzględnić konieczność zbrojenia na te naprężenia. Zgodnie z Eurokodem 5, naprężenia rozwarstwiające powinny być sprawdzane w strefie kalenicowej, której szerokość jest równa wysokości belki w kalenicy, a w przypadku przekroczenia dopuszczalnych wartości tych naprężeń, należy zaprojektować odpowiednie zbrojenie.

Eurokod 5 podaje szereg wzorów potrzebnych do obliczenia naprężeń normalnych prostopadłych do włókien. Ze względu jednak na to, że rozkład naprężeń rozwarstwiających jest w normie zdefiniowany z pewnym przybliżeniem, w wielu przypadkach konieczna staje się właściwa weryfikacja przy użyciu bardziej szczegółowych analiz. W niniejszej pracy proponuje się użycie metody elementów skończonych (MES) do badania belek klejonych z poprzecznym zbrojeniem. Autorzy skupili się na analizie naprężeń normalnych prostopadłych do włókien w belkach swobodnie podpartych zbrojonych poprzecznymi prętami, odnosząc je do belek bez zbrojenia. Praca wypełnia lukę w jakościowej analizie zbrojonych belek z drewna klejonego warstwowo. Opisana tutaj belka analizowana jest przy użyciu MES w ramach liniowej teorii sprężystości, z uwzględnieniem ortotropowych własności materiału. Głównym celem pracy jest ocena wpływu poprzecznego zbrojenia na wartości i rozkłady analizowanych naprężeń oraz identyfikacja najbardziej korzystnych schematów zbrojenia.

Przedstawiono dwa typy modeli numerycznych: model 2D belki w płaskim stanie naprężenia i model 3D. W obu przypadkach zbrojenie modelowane było jako elementy prętowe. Różnice pomiędzy modelami w przypadku belki bez zbrojenia kształtują się na poziomie 2÷6 % w zależności od badanego parametru, przy czym większe wartości uzyskano w modelu 3D. Model 2D okazał się wystarczający do jakościowej oceny wpływu zbrojenia na naprężenia rozwarstwiające, natomiast w modelu 3D ujawniły się koncentracje naprężeń wewnątrz belki, co ma duże znaczenie w przypadku zbrojenia dwoma rzędami prętów.

Symulacja numeryczna wykazała, że możliwa jest redukcja naprężeń rozwarstwiających w środku belki o 30 % przy użyciu prętów zbrojeniowych 6ϕ13. Należy jednak zwracać szczególną uwagę na rozkład naprężeń w okolicach dolnych końców prętów zbrojeniowych, gdyż mogą się tam pojawić maksymalne wartości naprężeń.

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